A Meshfree Simulation of the Draw Bending of Sheet Metal

Kalilou Sidibe, Guangyao Li

Abstract—The simulation of the draw bending of sheet metal is done using the Reproducing Kernel Particle Method (RKPM). The particle to segment contact algorithm is used for the contact detection as well as the contact constraints implementations. The penalty method is used for the implementation of the impenetrability condition. Both two-dimensional (2D) and three-dimensional (3D) draw bending of sheet metal are successfully simulated. The results obtained prove the effectiveness of the RKPM and the particle to segment contact algorithm for the sheet metal forming analysis.

Index Terms: Reproducing Kernel Particle Method (RKPM), meshfree methods, contact algorithms, draw bending, sheet metal forming

1 Introduction

With the development of mechanical science and engineering applications, in addition to the increasing use of computer in mechanics, numerical methods like the finite elements method (FEM), and more recently the meshfree (or meshless) methods have been the subjects of intensive researches for contact problems analysis. Among the meshfree methods are: Smoothed Particle Hydrodynamics (SPH) [1], [2] ; Diffuse Element Method (DEM) [3] ; Element-Free Galerkin (EFG) [2], [4], [5], [6] ; Reproducing Kernel Particle Method (RKPM) [7], [8], [9], [10], [11]; Partition of Unity Method (PUM) [12] and so forth.

The sheet metal forming is very an important process in manufacturing industries. The sheet metal forming requires the contact between tools (punch, blank holder, dies, …) and the blank (sheet metal).The numerical simulation of contact between different bodies, or different parts of a body is a challenging task in many engineering application. It requires the development of contact algorithms. Contact algorithms may be classified into two categories, contact searching algorithm and contact constraints algorithm. The contact searching consists of finding the contacting boundaries (contacting particles/nodes). The contact constraints algorithm is concerned with the implementation of the so-called impenetrability condition which does not allow overlapping between bodies, in other words two bodies cannot occupy the same space at the same time.

In the framework of FEM, several contact searching algorithm have been developed. These contact searching algorithms include the master-slave contact algorithm [13], [14]; the single surface contact algorithm [13], [14], the hierarchy territory contact algorithm [15], and the pinball contact algorithm [16], [17]. In the framework of meshless method, the particle to particle contact algorithm [18], [19], and the meshfree contact-detection algorithm [11], [20] were developed.

Designed to simulate high velocity impact of particles, the particle to particle contact algorithm represents the boundaries coarsely by particles (circular) therefore can not evaluate correctly the interpenetration of contact bodies. The meshfree contact-detection algorithm detects the overlapping between contacting bodies based on the determinant of the moment matrix, but does not allow direct evaluation of the amount of the interpenetration of the contact bodies. The accurate evaluation of the interpenetration is closely linked to the correct representation of the boundaries. These algorithms need an additional effort to represent correctly the boundaries in order to evaluate the gap between contacting bodies. Therefore, instead of using these algorithms, a '*particle to segment contact algorithm*' was developed and validated by G. Li et al. [21] for three-dimensional (3D) problems and K. Sidibe et al. [22] for two-dimensional (2D) problems. In the particle to segment contact algorithm the boundaries of the bodies are represented by particles located on the boundary, and these particles are interconnected to form polygons fitting the boundary without overlapping. The algorithm is originally developed for the simulation of the contact between a flexible body and several rigid bodies as encountered in metal forming where the workpiece is deformable and the tools are usually assumed to be rigid. This algorithm has the advantage to allow the correct evaluation of the interpenetration used for the determination of the contact constraints. K. Sibibe et al. [23] updated it to the simulation of flexible bodies contact as well as the self contact

2 RKPM Discretization

The reproducing kernel particle method (RKPM) is systematically formulated in [7], [8], [9], [10], [11]. The RKPM uses the finite integral representation of a function u($\mathbf{x}$) in a domain Ω_x.

$$u^a(\mathbf{x}) = \int_{\Omega x} \Phi_a(\mathbf{x}-\mathbf{y})u(\mathbf{y})d\Omega_x \qquad (1)$$

where $u^a(\mathbf{x})$ is the approximation of function u($\mathbf{x}$), $\Phi_a(\mathbf{x}-\mathbf{y})$ is the kernel function with compact support a.

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Discretizing the domain Ω_x by a set of particles $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{NP}\}$, where $\mathbf{x}_I$ is the position vector of particle **I**, and NP is the total number of particles; the integral is approximated by the following summation:

$$u^h(\mathbf{x}) = \sum_{I=1}^{NP} \mathbf{N}_I(\mathbf{x}) u(\mathbf{x}_I) \qquad (2)$$

where $\mathbf{N}_I(\mathbf{x})$ is the RKPM shape function defined to be

$$\mathbf{N}_I(\mathbf{x}) = \mathbf{C}(\mathbf{x}; \mathbf{x}-\mathbf{x}_I)\Phi_a(\mathbf{x}-\mathbf{x}_I)\Delta\mathbf{V}_I \quad (3)$$

$\mathbf{C}(\mathbf{x}; \mathbf{x}-\mathbf{x}_I)$ is the correction function introduced to improve the accuracy of the approximation near the boundaries and $\Delta\mathbf{V}_I$ is the volume of particle I and the subscript h is associated with a discretized domain.

The application of the principle of virtual work and the particles approximation to the equation of the conservation of the linear momentum leads to the equation of motion for contact problems (G. Li et al. [13]):

$$\mathbf{f}_I^{ext} - \mathbf{f}_I^{\text{int}} + \mathbf{f}_I^{cont} = \mathbf{M}_{IJ}\ddot{\mathbf{d}}_J \qquad (4)$$

Where:

- $\mathbf{f}_I^{ext}$: the external force of particle I
- $\mathbf{f}_I^{\text{int}}$: the internal force of particle I
- $\ddot{\mathbf{d}}_J$: the generalized acceleration of particle J
- $\mathbf{f}_I^{cont}$: the contact force of particle I (see **part III**)
- $\mathrm{M}_{IJ} = \int_{\Omega_0} \rho_0 N_I N_J d\Omega$ is the consistent mass matrix which can be approximated by row sum technique.

3 Particle to Segment Contact Algorithm

In this work, the particle to segment contact algorithm is used for the contact simulation. The particle to segment contact algorithm was developed by G. Li et al [21] for 3D problems and by K. Sidibe et al. [22] for 2D problems. Designated for the simulation of the metal forming analysis, the particle to segment contact algorithm modelled the tools as rigid bodies and the workpiece as flexible (deformable) body.

In 2D the boundaries of the rigid tools are discrtized by piecewise linear segments while in 3D the boundaries of tools assumed to be rigid are modelled by flat segments.

Every time step, prior to the calling to the **contact-subroutine**, the trial accelerations, velocities and displacements are computed from the explicit time routine. The trial positions of the particles, obtained from the trial displacements, are then used to check whether there is overlapping between the bodies. Whenever any overlapping is found the contact forces are evaluated and applied to cancel the interpenetration of the bodies as shown in **Figure 1**.

In **Figure 1**: $\mathbf{x}_S(t)$ is the position of S at time t (before penetration), $\tilde{\mathbf{x}}_S(t+\Delta t)$ trial position of S at time $t+\Delta t$ and $\mathbf{x}_S(t+\Delta t)$ corrected position of S at time $t+\Delta t$ (after application of contact forces); $\mathbf{n}$ is the normal unit vector of the contact segment pointing out from the segment toward the flexible body, $\mathbf{e}_1$ and $\mathbf{e}_2$ are the tangential unit vectors on the edges of the segment and g_N is the normal gap.

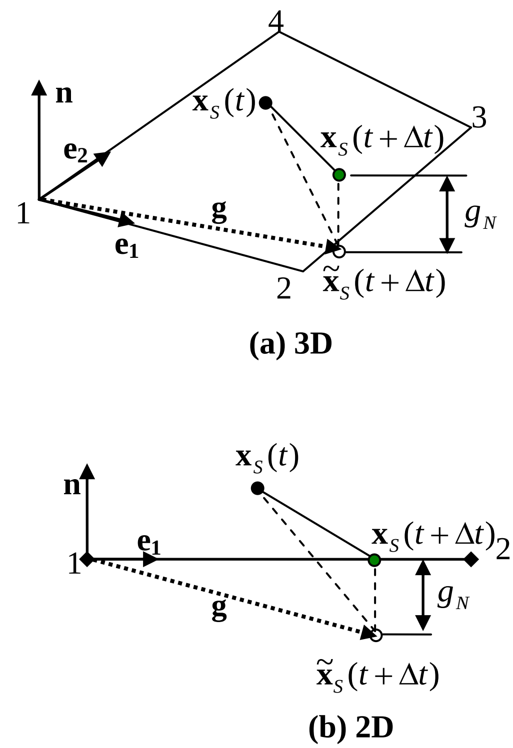


Fig. 1. Correction of trial position of the penetrating slave particle S

As shown in **Figure 1**, the normal gap is given by:

$$g_N = \mathbf{g}\cdot\mathbf{n} = (\tilde{\mathbf{x}}_S - \tilde{\mathbf{x}}_1)\cdot\mathbf{n} \qquad (5)$$

For each penetrating slave particle, the penalty force necessary to cancel the penetration is evaluated by:

$$\mathbf{f}_N(s) = -\frac{\mathrm{m}_s g_N}{\Delta \mathrm{t}^2}\mathbf{n} = f_n.\mathbf{n} \qquad (6)$$

The coulomb friction model is adopted to evaluate the friction between the contacting bodies. Frictional force applied to oppose the relative tangential displacement at the contact interface is given by:

$$\mathbf{f}_T(s) = -\min\left(\mu f_n, \left\|\frac{\mathrm{m}_s}{\Delta \mathrm{t}}\mathbf{v}_r\right\|\right)\frac{\mathbf{v}_r}{\|\mathbf{v}_r\|} \qquad (7)$$

Where μ is the friction coefficient on the contact interface and $\mathbf{v}_r$ is the tangential component of the relative velocity of the slave particle S with respect to the contact segment.

The resultant of the contact forces, on a given slave particle J is calculated by

$$\mathbf{f}_J = \mathbf{f}_N(J) + \mathbf{f}_T(J) \quad (8)$$

The force vectors calculated above are the exact nodal force vectors for each penetrating particle, to satisfy the impenetrability and friction conditions at the interface. Therefore the exact nodal force is re-distributed to a non-local 'fictitious force'. The fictitious force vector for a particle I is calculated as follows.

$$\mathbf{f}_I^{cont} = \sum_J \mathbf{f}_J N_I(\mathbf{X}_J) \quad (9)$$

4 Numerical Applications

Our current RKPM computer codes were tested and validated through standard test by G. Li et al. [21] for 3D formulation and K. Sidibe et al. [24] for the 2D formulation. Also the 3D implementation of the particle to segment contact algorithm was validated by G. Li et al. [21] and the 2D implemented by K. Sidibe et al. [22] for bulk metal forming. This work deals with the extension of these codes to the sheet metal forming simulation, the basic formulation staying the same. Here the standard problem of draw bending is treated for both 2D and 3D problems.

The geometry of the problem is described in **Figure 2**. In the simulation, the tools are assumed to be rigid. The punch is moved downward with the prescribed velocity of 10 m/s. The maximum stroke of the punch at the end of the simulation is 70 mm. The blank holder and the die are fixed. The gap between the die and the blank holder is equal to the thickness of the blank (0.81mm). The motion of the pad is synchronized with that of the punch. A blank of 350×35×0.81mm^3 with an elastoplastic material with isotropic linear strain hardening model is used.

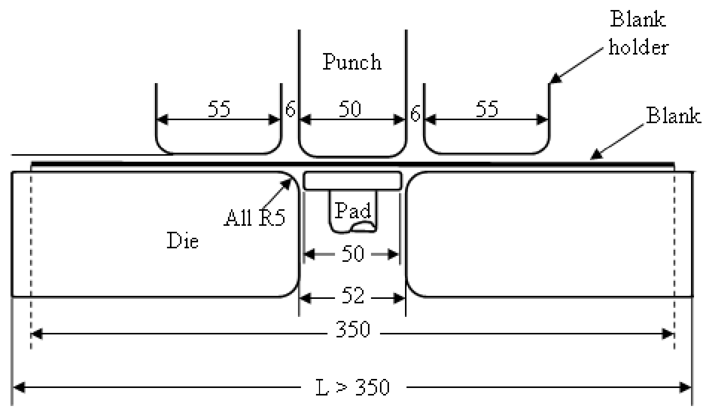
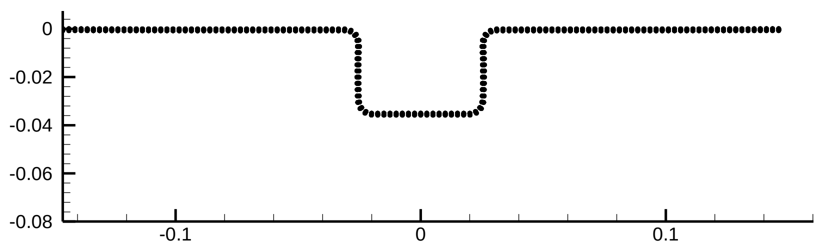


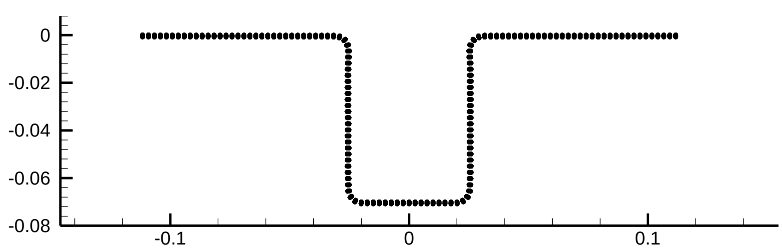
Fig. 2. Geometry description of the draw bending problem

4.1 Two-dimensional draw bending

A plane strain formulation is used in 2D. The blank of 350×0.81 mm^2 (in 2D) was discretized by 1755 (351×5) particles. The deformed shapes of the blank corresponding to two stages of the simulation of the sheet forming process in 2D are shown in **Figure 3**.



(a) Punch stroke of 35mm

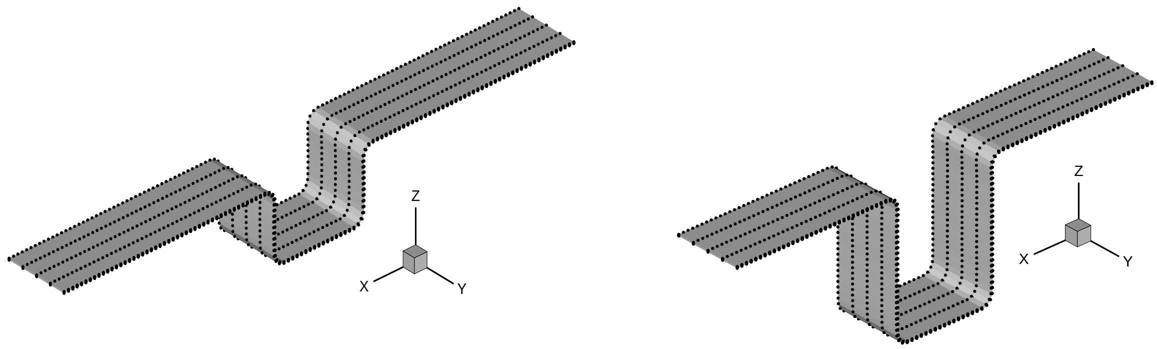


(b) Punch stroke of 70mm

Fig. 3. Deformed shape of the blank in 2D

4.2 Three-dimensional draw bending

A full 3D formulation is used for the simulation of the sheet forming process. The blank is discretized by a set of 3525 (141×5×5) particles. Two stages of the deformation of the blank are shown in **Figure 4**.



(a) Punch stroke of 35 mm (b) Punch stroke of 70 mm

Fig. 4. Deformed shape of the blank in 3D

5 Conclusion

The particle to segment contact algorithm, correctly implemented in the Reproducing kernel Particle Method, has been successfully used for the simulation of the draw bending of sheet metal. Both 2D and 3D problems are treated. A full 3D RKPM formulation has been used which has advantages than using shell theories as in the FEM in term of accuracy.

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